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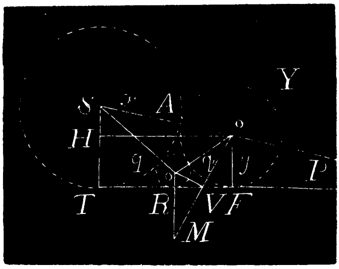


Solution.—Let P, a point in the tangent line, be the external center of similitude of the circles S and Y (radii x, y), then O_1, the center of Y_1 will be found by laying off $PR = PA$, drawing RM perpendicular to PT and equal to the radius y, as in the Figure, joining OM which will bisect RF in V. (F being the point of contact of circle Y and the tangent). A perpendicular to OM at V will meet RM in O_1.

Since q, q_1 are anti-homologous points on S and Y, $Pq \times Pq_1 = PA^2 = PR^2$, $\therefore$ R is the point of tangency of Y_1 and the tangent, and as OO_1M is an isosceles triangle, it is evident that $O_1R = O_1q_1$. A similar construction will show that Y_1 touches S.

Again, by drawing OH parallel to the tangent, and remembering that $TF = 2\sqrt{(xy)}$, it is easily found from the similar triangles HSO, TSP, that

$$PF = \frac{2y\sqrt{(xy)}}{x - y} \text{ and } PT = \frac{2x\sqrt{(xy)}}{x - y};$$

but F, T, being anti homologous points $PF \times PT = PR^2$.

$$\therefore \quad PR = \frac{2xy}{x - y}, \; RV = \tfrac{1}{2}(PR - PF) = \frac{xy - y\sqrt{(xy)}}{x - y}.$$

In the right triangle O_1VM, $VR^2 = RM \times O_1R$ or O_1R $(= y_1)$

$$= \frac{y[x - \sqrt{(xy)}]^2}{(x - x)^2}.$$

If the ratio of S to Y is a^2, $ay = x$ and

$$y_1 = \frac{x}{(\sqrt{a} + 1)^2} \text{ or } x = y_1(\sqrt{a} + 1)^2.$$

By putting $(\sqrt{a} + 1)^2$ for a in the last expression, $x = y_2(\sqrt{a} + 2)^2$, and by induction $x = y_n(\sqrt{a} + n)^2$; $\therefore$ $S = Y_n(\sqrt{a} + n)^4$.

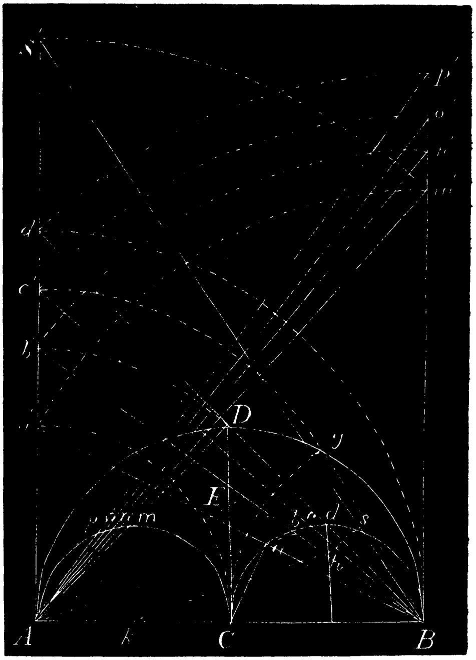
The radii y, y_1 &c. are to each other, inversely as the quantities a, $(\sqrt{a} + 1)^2$, $(\sqrt{a} + 2)^2$, &c. Hence, if $a = 1$, the square roots of these quantities form the natural series 1, 2, 3, 4, &c.

GEOMETRICAL PROBLEM AND SOLUTION.

BY T. P. STOWELL, ROCHESTER, N. Y.

Problem.—To divide a square into any number of equal squares.—Or to construct a square which shall equal any fractional part of a given square.

Solution.—Let AB be the side of any given square, upon which describe a semicircle, and on each radius AC and CB, describe other semicircles as shown in the diagram.



From B as centre and radius BC describe the arc Cg. Join Ag cutting the radius CD in E. Then, assuming $AB = 1$, we have by similar triangles $Ag^2 : AB^2 :: AC^2 : AE^2$ or $1 - \frac{1}{4} : 1 :: \frac{1}{4} : \frac{1}{3}$ Hence the square described on $AE = \frac{1}{3}$ the square on AB.

Again, from A as centre and radius AC describe the arc Ca'—Aa' being perpendicular to AB. Join Ba' cutting the semicircle on CB in a. Then because $Ba'^2 : AB^2 :: CB^2 : Ba^2$, therefore $1 + \frac{1}{4} : 1 :: \frac{1}{4} : \frac{1}{5}$; or the square desribed on $Ba = \frac{1}{5}$ the square on AB.

In a similar manner, $Bb = \frac{1}{6}$, $Bc = \frac{1}{7}$, $Bd = \frac{1}{8}$, $Am = \frac{1}{9}$, $An = \frac{1}{10}$, $Ao = \frac{1}{11}$, $Ap = \frac{1}{12}$ &c., of the square on AB; and there is a regular law governing the construction as shown in the diagram.

Again, suppose we wanted to construct a square $= \frac{7}{13}$ of a given square. From the above construction we know that $Bs^2 = \frac{1}{13}$ of AB^2 and $Bc^2 = \frac{1}{7}$ of AB^2. With radius Bs describe the arc sh cutting Bc in h. Join Ac and draw through h a line parallel with Ac cutting AB in k; then $Bk^2 = \frac{7}{13}$ of AB^2. Because by similar triangles $Bc^2 : BA^2 :: Bh^2 = Bs^2 : Bk^2$, or, multiplying the antecedents by 7, $7Bc^2 : BA^2 :: 7Bh^2 : Bk^2$. But by construction $7Bc^2 = BA^2$; $\therefore 7Bh^2 = Bk^2$, and because $Bh^2 = \frac{1}{13}$ of $BA^2 \therefore Bk^2 = \frac{7}{13}$ of BA^2, and similarly for any other fractional part.

It is evident that a circle, or any regular figure, may be divided in similar fractional parts by the same construction.

SOLUTION OF A PROBLEM.

BY E. B. SEITZ, GREENVILLE, OHIO.

To determine the radii of three circles inscribed in a triangle whose sides are a, b, c, each cir. touching the other two, and also two sides of the triangle.